

Deduction of the De Broglie's relation $\lambda=h/p$ from the classical electrodynamics



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Abstract

It is well known that De Broglie enables the use of the relativistic mechanics and –depending on a number of assumptions– to discover his relation $\lambda=h/p$, which led further to the creation of the theory of quantum mechanics (QM). However, after De Broglie's approach famous contradictions have appeared between De Broglie's theory and the Special Relativity Theory, also there were a number of suggestions in order to solve these contradictions. According to the suggested method in our papers it was remarked a serious part of the contradictions, and by following this method, we will present a new approach to derive the relation $\lambda=h/p$, starting from classical electrodynamics without any contradictions between Special Relativity Theory (SRT) and De Broglie's theory.

Keywords: De Broglie wave mechanics, Lorentz force Law, special relativity theory.

Resumen

Es conocido que De Broglie permite el empleo de la mecánica relativista y –dependiendo de un número de suposiciones– descubrir su relación $\lambda=h/p$, que condujo más adelante a la creación de la teoría de la mecánica cuántica (QM). Sin embargo, después del famoso enfoque de De Broglie han aparecido contradicciones entre la teoría de De Broglie y la teoría de la Relatividad Especial, también hubo sugerencias para resolver estas contradicciones. Según el método sugerido en nuestros artículos fue remarcada una parte seria de las contradicciones, y siguiendo este método, presentaremos una nueva aproximación para obtener la relación $\lambda=h/p$, partiendo de la electrodinámica clásica sin ninguna contradicción entre la Teoría de Relatividad Especial (SRT) y de la teoría de De Broglie.

Palabras claves: Mecánica ondulatoria de De Broglie, Ley de fuerza de Lorentz, teoría especial de la relatividad.

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I. INTRODUCTION

The development of QM was introduced in 1900 by Max Planck in his new hypothesis, namely: energy exchange between resonator and radiation takes place only in integer multiples of hf , where h is a new fundamental constant. This was known as the Planck's hypothesis for the quantization of the black body's emission

$$E = nhf, \quad n=1, 2, \dots \quad (1)$$

Five years later Einstein was able to generalize the Planck's hypothesis from the black body's emission to the electromagnetic field, he described it according to small particles (photons), which are distinct by frequency, and every photon carries the energy

$$E = hf. \quad (2)$$

However, Eq. (2) contradicts with many experiments which were explained using the wave-theory for the electromagnetic field. Nevertheless, Einstein was able to explain the photoelectrical effect using Eq. (2) after the fail of wave-theory in explaining it. Later, Planck's hypothesis for the quantization didn't stop on the electromagnetic field's

limit, in no time the need for this hypothesis for the electron emerged.

II. BOHR'S QUANTIZATION HYPOTHESIS FOR THE ATOMIC ORBITAL

N. Bohr extended this hypothesis to contain an electron and to give a curing explanation for the hydrogen atom spectrum, whose analysis showed that only light at certain definite frequencies and energies were emitted, and the experiments show that the spectrum of the hydrogen atom contains a separated line, and satisfies this equation

$$f \propto \left[\frac{1}{n^2} - \frac{1}{m^2} \right], \quad (3)$$

where m, n are hydrogen atom's levels. The classical theory of emission was unable to explain Eq. (3), this led Bohr to postulate that the circular orbit of the electron around the nucleus is quantized, that is, its angular momentum could only have certain discrete values, these being integer multiples of a certain basic value. First, let us follow the usual pathway where Bohr's quantization is introduced. Bohr

started from the hypothesis which says that the electron of the hydrogen atom moves in a circular orbit around the nucleus, and in this way the potential energy is the electrostatic energy

$$V = -\frac{e^2}{4\pi\epsilon_0 r}, \quad (4)$$

where r is the radius of the electronic orbit. Using Newton's second law for the electron moving in a circular orbit, and thus subject to Coulomb's law, we have

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}, \quad (5)$$

this allows us to calculate the kinetic energy of the electron in such an orbit

$$T = \frac{1}{2}mv^2 = \frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r}, \quad (6)$$

therefore, the total energy for the system is

$$E = T + V = -\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r}. \quad (7)$$

Substituting Eq. (2) in Eq. (3), we write

$$E = R \left[\frac{1}{n^2} - \frac{1}{m^2} \right], \quad (8)$$

where R is the Rydberg constant. Comparing Eq. (8) with Eq. (7) we find

$$E \propto \frac{1}{n^2}, \quad (9a)$$

$$r \propto n^2, \quad (9b)$$

Eq. (8) implies that the change in energy from the level n to the level m causes an emission or absorption of the electromagnetic energy as photons, as well as Eq. (9a) means that the energy levels of the electron are quantized, where Eq. (9b) says that the electron can move only on non-continuous specified orbits. All these contradict the classical theory of emission.

Let's now see how the momentum quantization relation of the electron is derived. Eq. (5) indicates that

$$mvr \propto \sqrt{r}. \quad (10)$$

To let this equation agree with the experimental information the proportionality constant was taken on the form $\hbar=h/2\pi$, and by substituting Eq. (9b) in Eq. (10) we then have

$$mvr = n\hbar. \quad (11)$$

We see that the analysis of the hydrogen spectrum, Eq. (3), plus the Planck- Einstein hypothesis of energy, Eq. (1), leads that the angular momentum of the electron is quantized, Eq. (11). The importance of Eq. (11) emerges from the generalization of the quantization over matter, after it was monopolized on energy.

III. SPECIAL RELATIVITY AND DE BROGLIE THEORY

After the creation of the electromagnetic theory of light, it become possible to formulate the laws of the corpuscular properties of radiation and the wave properties of the corpuscular as

$$p = h/\lambda = \hbar k, \quad k = 2\pi/\lambda, \quad (12a)$$

as well as

$$E = hf = \hbar \omega, \quad p = h/\lambda = \hbar k. \quad (12b)$$

It was clear to De Broglie [1, 2] that the electromagnetic field demonstrates particles properties after it was demonstrating waves properties according to wave-theory, That's why De Broglie wanted to create a theory which contains both properties of wave and its particle counterpart in light, and he understood Eq. (12b) as following: The quantities E and p specify the properties of a particle (photon), while the quantities λ and ω specify a wave properties, which means that Eq. (12b) connects between the wave and particle properties of the light, and he asked himself why we don't generalize Eq. (12b) for an electron and getting Eq. (11) as a result. So De Broglie, postulated the validity of relation (12b) for a particle with rest mass m_0 through his hypothesis of the "periodic phenomenon", *i.e.*:

$$hf_0 = m_0 c^2. \quad (13)$$

The frequency f_0 is to be measured, of course, in the rest frame of the particle; with Eq. (13) De Broglie connected between two different things: The left side shows that the matter is a type of energy, while the right side shows that the energy is a type of matter. The starting point for De Broglie was applying the Special Relativity Theory (SRT) [3] on Eq. (13), since he considered that if it was correct for a rest particle then it must be correct for a moving particle. So Eq. (13) is correct for the rest frame S' related to the particle, and for an observer in the frame S which is in uniform motion with constant velocity u and $u \parallel Ox$, then Eq. (13) must have the form

$$hf = mc^2. \quad (14)$$

De Broglie, assumed that $E = hf$ does hold for the relativistic electron. On the other hand, the relation $E = m_0 c^2 (1 - v^2/c^2)^{-1/2}$ for the electron in SRT implies that E increases with the velocity, so does, the frequency of moving electron yields an increased frequency

$$f = \frac{f_0}{\sqrt{1 - \frac{v^2}{c^2}}}. \quad (15a)$$

However, as is well known in SRT, if the clock has a frequency f_0 in the rest frame of particle, its frequency, according to the so-called time dilation, when it is moving at a velocity v in frame S

$$f = f_0 \sqrt{1 - \frac{v^2}{c^2}}. \quad (15b)$$

Evidently, Eq. (15b) is just opposite to Eq. (15a), indeed accounting for time dilation leads to slow down "moving clock" frequency. Thus, it is clear that some additional assumption is needed to overcome such a fundamental contradiction. To find the way out of this paradox De Broglie assumed that f in Eq. (15b) is not the frequency of a clock moving with the particle, but the frequency of a wave accompanying the particle propagating with velocity v_p in the direction of motion. The fact that its velocity $v_p = c^2/v$ is necessarily greater than the light speed c , shows that it can not represent transport of energy.

IV. DE BROGLIE METHOD TO DERIVE THE RELATION $\lambda=h/p$

In his work [1] De Broglie stress the importance of Fermat's as well as Hamilton's principles, from which, the well-known de Broglie relation $\lambda=h/p$, as a consequence.

We shall see now how he matching between the Fermat's principle and the principle of Hamilton to derive his well-known relation $\lambda=h/p$.

De Broglie have found from the study of the mechanics and wave propagation that the application of the Fermat's principle on the phase wave φ

$$\delta \int_A^B d\varphi = 0, \quad (16)$$

similar to the application of the Hamilton's principle on the moving particle:

$$\delta S = \delta \int_A^B L(q, \dot{q}, t) dt = 0, \quad (17)$$

where L is the Lagrangian of a relativistic electron moving in electromagnetic fields described by the vector potential A and the scalar potential V .

$$L = -m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}} + e A \mathbf{v} - e V. \quad (18)$$

The matching between the Fermat's principle and the principle of Hamilton had not been achieved by De Broglie in only writing both relations (16) and (18) in the 4-d form. We consider now the matter of relativistic dynamics for an electron. If we have the metric in the form $ds^2 = (dx^4)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$, where space coordinates are labeled x^1 , x^2 and x^3 , the coordinate ct is denoted by x^4 . Then the 4-d velocity u^μ , of an electron is

$$u^\mu = \frac{dx^\mu}{ds}, \quad \mu = 1, 2, 3, 4.$$

In modern physics, the relativistic postulates that were presented by Einstein became the base to represent the relativistic transformation equation. According to Einstein, the mathematical equations expressing the laws of Nature must be covariant, that is, invariant in form when we make a Lorentz transformation of the coordinates. In the 4-d formulation we ensure covariance of equations (same form

in all inertial frames) by writing them in terms of Lorentz scalars, 4-vectors, tensors. We consider an electron with proper velocity u^μ moving into the 4-d electromagnetic fields described by

$$A^\mu = (A^4, A^j) = \left(\frac{V}{c}, -A^x, -A^y, -A^z \right).$$

The Lagrangian of a relativistic electron, Eq. (18) in 4-d form is:

$$L = -c \sqrt{1 - \frac{v^2}{c^2}} \left[m_0 c + e A_\mu u^\mu \right],$$

or

$$L dt = -c \left[m_0 c + e A_\mu u^\mu \right] ds, \\ = -c \left[m_0 c u_\mu + e A_\mu \right] dx^\mu.$$

De Broglie then defined the 4-d vector J_μ by the relation $J_\mu = m_0 c u_\mu + e A_\mu$, the statement of least action in Eq. (17) then gives:

$$\delta \int_A^B J_\mu dx^\mu = 0. \quad (19)$$

We shall study now phase wave propagation using a method parallel to that of the principle of Hamilton. To do so, we take phase wave depending on space-time coordinates x^μ . Writing also $d\varphi$ in the relationship (16), according to the 4-d form

$$d\varphi = \frac{\partial \varphi}{\partial x_\mu} dx^\mu,$$

where $\varphi = \hbar(\omega t - k r)$. Thus the Fermat's principle, Eq. (16), becomes according to the 4-d form the following

$$\delta \int_A^B \frac{\partial \varphi}{\partial x_\mu} dx_\mu = 0. \quad (20)$$

By comparing the spatial and temporal parts of relations (19) and (20) we find

$$J_4 = \frac{\partial \varphi}{\partial x_4}, \quad J_i = \frac{\partial \varphi}{\partial x_i}. \quad (21)$$

However, De Broglie gave the physical interpretation of the 4-d vector J_μ as energy-momentum vector

$$J_4 = \frac{E}{c}, \quad J_i = -p_i. \quad (22)$$

Thus, if we apply Eq. (21) to the phase wave $\varphi = \hbar(\omega t - kr)$, we have

$$\frac{\partial \varphi}{\partial x_4} = \frac{\partial \varphi}{\partial (ct)} = \frac{\omega \hbar}{c}, \quad \frac{\partial \varphi}{\partial x_i} = -\hbar k.$$

Now from Eq. (22), and by taking into account the last relation, we obtain

$$\frac{E}{c} = \frac{\hbar\omega}{c} \Rightarrow E = \hbar\omega, \quad (23a)$$

$$-p_i = -\hbar k \Rightarrow p = \hbar k, \quad (23b)$$

where $p = mv$. Eqs. (23) are relations of the electron wave just as the relations (12) are relationships of the wave of photon. In particular, the relation (23b) is the famous De Broglie's relation

$$p = \hbar k \Rightarrow \lambda = \frac{h}{p}. \quad (24)$$

Eq. (11) can be derived now as a result of using the famous De Broglie's relation, Eq. (24). For all physical systems whose coordinates are periodic functions of time there is a quantum condition for each coordinate expressed as

$$\oint p dr = nh.$$

In our case, we have $p = -\nabla\varphi = \hbar k$, i.e.,

$$\hbar k \oint dr = nh.$$

For the electron moving in a circular orbit around the nucleus

$$\hbar k l = n h \Rightarrow \frac{h}{2\pi} \frac{2\pi}{\lambda} l = n h \Rightarrow l = n \lambda,$$

and by using Eq. (23b) and $l=2\pi r$, we get $r \cdot p = nh/2\pi$, that is, $L = r \cdot p = n\hbar$, obtaining therefore the same result of Bohr, Eq. (11).

V. NEW METHOD TO DERIVE THE RELATION $\lambda=h/p$ FROM CLASSICAL ELECTRODYNAMICS

In 1925, L. De Broglie proposed the idea of matter waves, which was that any particle is associated with a so-called pilot wave: The momentum of one and the wave-vector of the other are proportional and the coefficient of proportionality is a universal constant. Any particle of the 4-d energy-momentum $p_\mu = (E/c, \mathbf{p})$ is "associated with" a wave of 4-d wave-vector $k_\mu = (\omega/c, \mathbf{k})$ proportional to p_μ , and this 4-d equality breaks down into a scalar component and a vectorial component usually stated using only the magnitude p of the vectorial momentum $p = |\mathbf{p}|$, namely: $E = \hbar\omega$, $p = \hbar k$. The above relations make the phase wave v_p equal to E/p . For a particle of rest mass m_0 and mechanical velocity v , we have $E^2 = c^2 p^2 + c^4 m_0^2$, therefore $1/v_p^2 = (v/c^2)^2$. This establishes an extremely simple relation between the phase wave and mechanical velocity as $v_p v = c^2$. Since the appearance of De Broglie theory, which was formulated through SRT's relations, an obviously contradictions between them was raised.

Wave mechanics, now a fundamental part of quantum theory, does not allow for interpretation of the wave function as a physical (real) wave, due to the difference between phase velocity v_p and group velocity $v_{gr} = v$ of de Broglie waves. This situation has been the subject of much scientific

controversy, discussion, and attempts at resolution. The first of these attempts can be attributed to J. P. Wesely [4], who supposed a real wave function instead of the complex wave function in traditional quantum theory. And he could prove that the phase velocity equals the particle velocity. Another attempt in this context is by M. Wolff [5], he analyzes a spherical wave structure for the moving electron, and he formulates SRT free from the usual contradiction, then he concludes the compatibility between SRT and de Broglie theory. Recently, R. Ferber [6] has showed that the following relation $v_p v = c^2$ is a result of using the Lorentz transformation, and not a result of de Broglie's hypothesis. A. P. Kirilyuk [7] also derived all major laws and physical entities, including De Broglie version and relativistic dynamics, as intrinsically unified manifestations of the underlying complex-dynamic interaction process.

In several recent papers [8, 9] we showed that choosing new sets of postulates, including classical (pre-Einstein) physics laws, within the main body of the SRT and applying the classical relativity principle, enables us to cancel the Lorentz transformation from the main body of SRT. And these enable us to derive all the famous dynamic equations for the charged particle like the relativistic mass and to derive Einstein's equation $E = mc^2$ from classical physical laws.

As a result de Broglie relations were deduced from classical physical laws and emphasize the incompatibility between SRT and particle dynamics [10, 11, 12]. These incompatibilities arise because the Lorentz transformation and its kinematical effects have the primacy over the physical law in deriving the relativistic dynamical quantities. As in the paper [8] we present a derivation of relativistic Lagrangian starting from classical Lorentz force, without calling upon the usual approaches in relativistic mechanics

$$L = -m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}} + e \mathbf{A} \mathbf{v} - e V. \quad (25)$$

Now we need to clarify that Eq. (25) will show the electron wave nature, without matching between Hamilton and Fermat principles and without writing Eq. (25) in 4-d form. As we know that Hamilton function in the classical mechanic is given by

$$H = v \frac{\partial L}{\partial v} - L. \quad (26)$$

Substituting Eq. (25) into Eq. (26) gives

$$H = v(mv + eA) + m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}} - evA + e\phi.$$

It can be proved easily that $mc^2 = mv^2 + m_0 c^2 (1 - v^2/c^2)^{1/2}$ (see the refs. [10, 11, 12]), so we have

$$H = mc^2 + e\phi, \quad (27)$$

and by squaring Eq. (27) we find

$$(H - e\phi)^2 = m^2 c^4 = c^2 p^2 + m_0^2 c^4. \quad (28)$$

Finally, by the differential of Eq. (28) gives $HdH = c^2 pdp$ or

$$dH = v dp, \quad (29)$$

It is well known that the Hamilton function represents energy $H = hf = \hbar\omega$, then Eq. (29) may be written as $d\omega = v dp/\hbar$; using the definition of the group velocity $v_g = v = d\omega/dk$ we find

$$dk = \frac{dp}{\hbar}, \quad (30)$$

and by integration of Eq. (30), considering the initial condition where $v = 0$ so $k = 0$, and will get the famous equation

$$k = \frac{p}{\hbar} \Rightarrow \lambda = \frac{h}{p}. \quad (31)$$

VI. CONCLUSION

It is true, in De Broglie's thesis that he started his analysis from SRT. Hence he starts from the Lagrangian for an electron moving in relativistic velocity to derive Eq. (24). We demonstrated that if we start from the same Lagrangian which has been derived from the classical electromagnetic base, it will give us the mentioned relation, Eq. (31).

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