



Reactance of a Parallel *RLC* Circuit

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Abstract

We study how the reactance Z of a parallel *RLC* circuit changes with driving frequency ω . We found that in the case that no resistor is connected to the L and C , there is a maximum Z appears at the same ω while the Z approaches to zero when $\omega \rightarrow 0$ and $\omega \rightarrow \infty$. However, in the case that a resistor is connected to the L and C respectively, a maximum or a minimum of Z can appear depending on ω/ω_0 where $\omega_0 = 1/\sqrt{LC}$.

Keywords: *RLC* Circuit, Reactance, Driving frequency.

Resumen

Estudiamos cómo cambia la reactancia Z de un circuito *RLC* paralelo al variar la frecuencia ω . Encontramos que en el caso de que no haya resistor conectado a L y C , hay un máximo de Z que aparece a la misma ω mientras que Z tiene a cero cuando $\omega \rightarrow 0$ y $\omega \rightarrow \infty$. Sin embargo, en el caso de que el resistor sea conectado a L y C respectivamente, puede aparecer un máximo o un mínimo de Z dependiendo en ω/ω_0 donde $\omega_0 = 1/\sqrt{LC}$.

Palabras clave: Circuito *RCL*, Reactancia, Variación de frecuencia.

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While *RLC* circuit has been extensively studied [1, 2, 3, 4], some confusions may still occur. We are attracted by this problem: In a parallel *RLC* circuit, how does the reactance change with driving frequency? The reason that we are attracted by it is that without numerical calculation, it is hard to get conclusion by doing qualitative analysis only.

We first consider the simplest parallel *RLC* circuit in which there is no resistor connected to L and C . Then we consider the case that there is a resistor R connected to L and C . The first case is shown in Fig. 1.

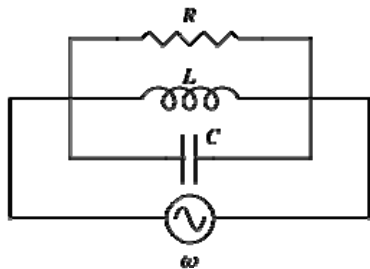


FIGURE 1. Schematic view of a simplest parallel *RLC* circuit. The driven frequency is ω , and voltage is any reasonable value.

A. Qualitative analysis

Consider two extremes at first: frequency ω is zero and infinity. When ω is zero, the impedance of L is zero so is the whole circuit; when ω is infinity, the impedance of C is zero so is the whole circuit. Therefore, the reactance of the circuit should approach to zero at both ends of ω approaching to zero and infinity.

But how about the reactance in other frequency? Where is the maximum if there is a one?

B. Quantitative analysis

The reactance of the whole circuit is

$$\begin{aligned} Ze^{i\phi} &= \left(\frac{1}{R} + \frac{1}{iX_L} - \frac{1}{iX_C} \right)^{-1}, \\ &= \left(\frac{1}{R} + \frac{1}{i\omega L} + i\omega C \right)^{-1}, \\ &= R \left(1 + \frac{R}{i\omega L} + iR\omega C \right)^{-1}. \end{aligned} \quad (1)$$

That is

$$\frac{Ze^{i\phi}}{R} = \left(1 + \frac{R}{i\omega L} + iR\omega C \right)^{-1}. \quad (2)$$

To write this in terms of dimensionless variables, we define:

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad \omega = \gamma\omega_0, \quad R_0 = \sqrt{\frac{L}{C}}, \quad \text{and} \quad \rho = \frac{R_0}{R}.$$

Thus,

$$\frac{Ze^{i\phi}}{R} = \left(1 - \frac{i}{\gamma\rho} + i\frac{\gamma}{\rho}\right)^{-1}. \tag{3}$$

Then the $\left|\frac{Ze^{i\phi}}{R}\right|$ gives the relative reactance value as a function of γ and ρ . Fig. 2 shows the relation between $\left|\frac{Ze^{i\phi}}{R}\right|$ and γ with various values of ρ . We can see that the trend of $\left|\frac{Ze^{i\phi}}{R}\right|$ agrees with our qualitative analysis that when γ is small and large, $\left|\frac{Ze^{i\phi}}{R}\right|$ approaches to zero. But the fact that location of the maximum of $\left|\frac{Ze^{i\phi}}{R}\right|$ is at $\gamma = 1$ cannot be predicted by qualitative analysis. It is surprising to see that regardless of the ρ values, $\left|\frac{Ze^{i\phi}}{R}\right|$ reaches to 1 when $\gamma = 1$.

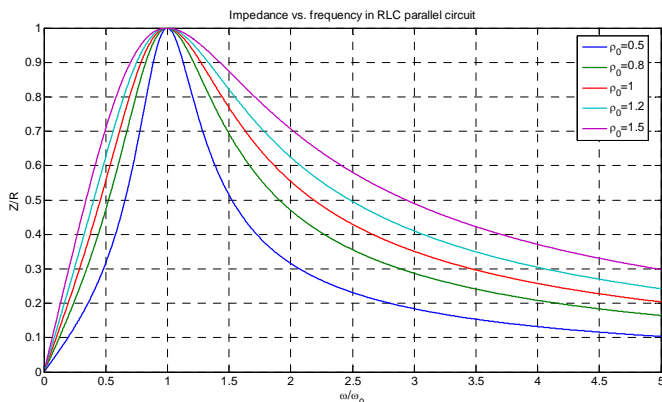


FIGURE 2. Relation between $\left|\frac{Ze^{i\phi}}{R}\right|$ and ω/ω_0 . $\left|\frac{Ze^{i\phi}}{R}\right|$ approaches zero when ω/ω_0 is zero and infinity, which is agree with our qualitative analysis.

Now we consider a scenario which is a little more complicated.

Suppose that there is a resistor R connected in series with L and C , which is shown in Fig. 3. How does the $\left|\frac{Ze^{i\phi}}{R}\right|$ change with ω/ω_0 ?

C. Qualitative analysis

Consider two extremes at first: frequency ω is zero and infinity. When ω is zero, the impedance of L is zero so the resultant reactance of the whole circuit is $R/2$. Or in another word, $\left|\frac{Ze^{i\phi}}{R}\right| = 1/2$. When ω is infinity, the impedance of C is zero so the resultant reactance of the whole circuit is $R/2$. Or in another word, $\left|\frac{Ze^{i\phi}}{R}\right| = 1/2$, again. Therefore, the reactance of the circuit should approach to $1/2$ at both ends of ω approaching to zero and infinity.

But how about the reactance in other frequency? Where is the maximum/minimum if there is a one?

D. Quantitative analysis

The reactance of the whole circuit is

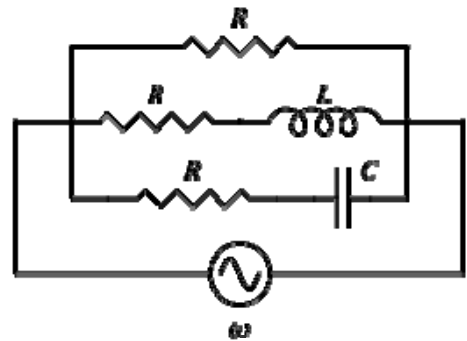


FIGURE 3. Schematic view of a parallel RLC circuit. Now there is a resistor connected to L and C in series. The driven frequency is still ω , and voltage is any reasonable value.

That is

$$\frac{Ze^{i\phi}}{R} = \left(1 + \frac{1}{1 + i\omega L/R} + \frac{1}{1 - \frac{i}{R\omega C}}\right)^{-1}. \tag{5}$$

To write this in terms of dimensionless variables, we get

$$\frac{Ze^{i\phi}}{R} = \left(1 + \frac{1}{1 + i\gamma\rho} + \frac{1}{1 - \frac{i\rho}{\gamma}}\right)^{-1}. \tag{6}$$

Then, the $\left|\frac{Ze^{i\phi}}{R}\right|$ gives the relative reactance value as a function of γ and ρ . Figure 4 shows the relation between $\left|\frac{Ze^{i\phi}}{R}\right|$ and γ with various values of ρ . We can see that the trend of $\left|\frac{Ze^{i\phi}}{R}\right|$ agrees with our qualitative analysis that

when γ is small and large, $\left| \frac{Ze^{i\phi}}{R} \right|$ approaches to $1/2$.

However, when $\rho < 1$, there is a minimum value; when $\rho > 1$, there is a maximum value. Both locate at $\gamma = 1$. When $\rho = 1$, $\left| \frac{Ze^{i\phi}}{R} \right|$ does not change with frequency.

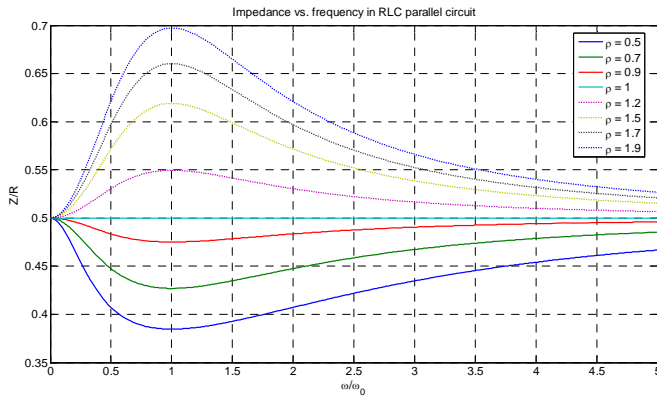


FIGURE 4. Relation between $\left| \frac{Ze^{i\phi}}{R} \right|$ and $\omega/\omega_0 \left| \frac{Ze^{i\phi}}{R} \right|$ approaches $1/2$ when ω/ω_0 is zero and infinity, which agrees with our qualitative analysis. However, there is a minimum when $\rho > 1$, which is different from the simplest parallel *RLC* circuit.

Reactance analysis on parallel *RLC* circuit is usually ignored in the teaching of introductory physics. Instead, response of reactance to frequency on series *RLC* circuit is taught to show the asymptotic behavior of *L* and *C*. Therefore, some students and teachers are confused when they see that *R*, *L*, and *C* are connected in parallel. In the numerical analysis, we show that the reactance of parallel *RLC* circuit can be complicated: if there is no resistor connecting to *L* and *C*, there is always a maximum value; but if there is a resistor connecting *L* and *C*, there can be a minimum value.

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