

Motion of an electric dipole in a static electromagnetic field



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Abstract

Expressions for the force and torque on a moving electric dipole in a magnetostatic field are combined with those due to an electrostatic field. By sending oriented molecular dipoles into a region of crossed magnetic and electric fields, the molecules are selectively rotated based on the direction and magnitude of their velocities. In principle this field configuration could be used to create a molecular isolator that only lets molecules through in one direction.

Keywords: Electric dipole moment, electromagnetic force and torque, molecular orientation.

Resumen

Combinamos expresiones para la fuerza y la torca en un dipolo eléctrico en movimiento en un campo magnetostático con las ocasionadas por un campo electrostático. Enviando dipolos moleculares orientados en la región de cruce de los campos magnético y eléctrico, las moléculas son rotadas selectivamente en la dirección y magnitud de sus velocidades. En principio esta configuración de campo podría ser usada para crear un aislador molecular que sólo permite moléculas en una dirección.

Palabras clave: Momento dipolar eléctrico, fuerza electromagnética y torca, orientación molecular.

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1 I. INTRODUCTION

2
3 A classical electric dipole $\vec{p} \equiv q\vec{L}$ consists of a positive
4 charge distribution $+q$ whose centroid is displaced by \vec{L}
5 relative to the centroid of a negative charge distribution $-q$.
6 (Quantum effects such as Stark mixing can induce dipole
7 moments in molecules which alter this simple picture [1].)
8 For example, in a gaseous NaCl molecule, the much greater
9 electronegativity of the chlorine atom as compared to that of
10 the sodium atom causes Cl to steal an electron away from
11 Na, resulting in an ionic bond between Cl^- and Na^+ . One
12 can thereby estimate [2] the magnitude of its dipole moment
13 to be the elementary charge e multiplied by the bondlength
14 L , giving $p \approx 4 \times 10^{-29} \text{ C}\cdot\text{m} = 12 \text{ D}$. Introductory physics
15 textbooks show that the torque on an electric dipole \vec{p} in an
16 electric field \vec{E} is $\vec{\tau}_{pE} = \vec{p} \times \vec{E}$. In addition, the motion of
17 point charges in electric and magnetic fields is discussed.
18 Similar ideas are used in the present article to discuss the
19 motion of an electric dipole in static magnetic and electric
20 fields.

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23 II. FORCES AND TORQUES

24
25 Label the point midway between the centers of positive and
26 negative charge of the dipole \vec{p} as O. (This point may or
27 may not coincide with the center of mass of the object.)
28 Decompose the motion of the dipole at any instant into a
29 translation of point O with linear velocity \vec{v} and a rotation
30 of the dipole about point O with angular velocity $\vec{\omega}$. Now
31 suppose the dipole enters a region of uniform magnetic field
32 \vec{B} . The magnetic forces on the two charge centers will be

$$\vec{F}_+ = q(\vec{v} + \vec{\omega} \times \vec{L}/2) \times \vec{B} \quad \text{and} \quad (1)$$

$$\vec{F}_- = -q(\vec{v} - \vec{\omega} \times \vec{L}/2) \times \vec{B}.$$

34 Consequently the net force on the dipole becomes the triple
35 vector product

$$\vec{F}_{pB} = (\vec{\omega} \times \vec{p}) \times \vec{B}, \quad (2)$$

37 while the torque about O is

$$\vec{\tau}_{pB} = \vec{p} \times (\vec{v} \times \vec{B}). \quad (3)$$

1 In the frame of reference of point O, the magnetic field is
 2 relativistically transformed into an electric field $\vec{E}' = \vec{v} \times \vec{B}$
 3 and thus $\vec{\tau}_{pB} = \vec{\tau}_{pE'}$ which seeks to rotate \vec{p} into the
 4 direction of \vec{E}' .

5 Suppose the dipole starts out with zero angular velocity
 6 but its translational velocity is perpendicular to the applied
 7 magnetic field so that the torque in Eq. (3) is maximized. To
 8 be specific, choose \vec{v} to define the $+x$ direction and \vec{B} the
 9 $+y$ direction. Now the torque is largest if \vec{p} lies in the xy
 10 plane. In that case $\vec{\tau}_{pB}$ will also lie in the xy plane and will
 11 be perpendicular to \vec{p} . As a result, the dipole will begin to
 12 librate (rock back and forth) end over end; that is, it will
 13 oscillate (indefinitely in the absence of drag) like a pendulum
 14 with the apex of its circular arc in the $+z$ direction.
 15 Associated with these rotational oscillations will be a
 16 periodically varying force \vec{F}_{pB} , alternately decelerating and
 17 accelerating the translations of the dipole because \vec{F}_{pB} is
 18 parallel to \vec{v} . In turn that force affects the torque by varying
 19 \vec{v} in Eq. (3), although the feedback will be weak if the mass
 20 of the dipole and/or its moment of inertia about O is large.

21 Incidentally, note that we can use a vector identity to
 22 rewrite Eq. (3) as

$$23 \quad \vec{\tau}_{pB} = (\vec{p} \times \vec{v}) \times \vec{B} + (\vec{B} \times \vec{p}) \times \vec{v}. \quad (4)$$

24 If the dipole moment is initially parallel to the magnetic
 25 field, then even after it begins to tumble, \vec{p} will always lie in
 26 the yz plane. Therefore $\vec{B} \times \vec{p}$ will be parallel to the x axis,
 27 and the last term in Eq. (4) will be zero. We can then
 28 interpret $\vec{p} \times \vec{v}$ as a magnetic dipole moment $\vec{\mu}$ in the $-z$
 29 direction. Charge $+q$ initially located at $y = L/2$ and
 30 traveling in the $+x$ direction is equivalent to a current I
 31 circulating clockwise as seen looking down along the z axis.
 32 Likewise charge $-q$ initially located at $y = -L/2$ and
 33 traveling in the $+x$ direction corresponds to the same
 34 clockwise current I . We thus have a current loop,
 35 corresponding to a magnetic dipole. Equation (4) can now be
 36 interpreted as $\vec{\tau}_{pB} = \vec{\tau}_{\mu B}$ where $\vec{\tau}_{\mu B} = \vec{\mu} \times \vec{B}$ is the torque
 37 on a magnetic dipole.

38 Next suppose that the region also contains a uniform
 39 electric field \vec{E} . The electric forces on the two charges
 40 constitute a couple, so that \vec{F}_{pE} is zero and $\vec{\tau}_{pE} = \vec{p} \times \vec{E}$. As
 41 a result, the overall electromagnetic force and torque on the
 42 electric dipole are

$$43 \quad \vec{F}_p = (\vec{\omega} \times \vec{p}) \times \vec{B} \quad (5)$$

44 and

$$45 \quad \vec{\tau}_p = \vec{p} \times (\vec{E} + \vec{v} \times \vec{B}), \quad (6)$$

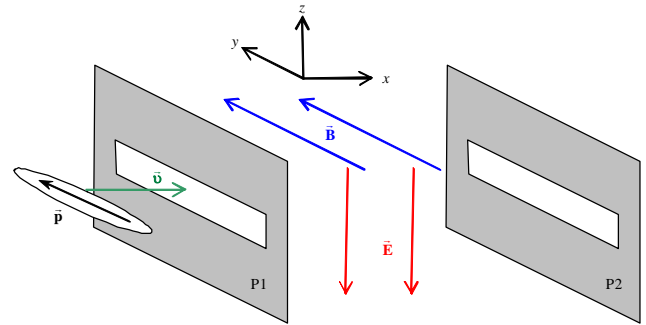
46 respectively. Equation (6) can be interpreted as the cross-
 47 product of \vec{L} with the Lorentz force. Now suppose we
 48 choose to cross the electric field with both the magnetic field

49 and translational velocity, $\vec{E} = \vec{B} \times \vec{v}$ (so that it is equal to
 50 $-\vec{E}'$ and points in the $-z$ direction). In that case, $\vec{\tau}_p$ is zero
 51 and the dipole will no longer begin to librate. This situation
 52 is the rotational analog of a “translational velocity selector”
 53 whereby a point charge entering a region of crossed electric
 54 and magnetic fields experiences zero net force.

57 III. MOLECULAR APPLICATION

58 As an application, one can imagine a device analogous to a
 59 Faraday optical isolator, as sketched in Fig. 1. Consider long,
 60 cigar-shaped molecules with one end negatively charged and
 61 the other end positively charged so that there is an electric
 62 dipole moment directed along their length (such as linear
 63 HCN trimers with $p = 11$ D [3, 4]). Suppose that a beam of
 64 them is incident on a horizontal molecular polarizer P1 that
 65 only transmits molecules oriented parallel to the y direction.
 66 (It might be possible to construct such a polarizer by milling
 67 nano-sized slits through an impermeable membrane [5].) The
 68 molecules enter a region with a magnetic field in the $+y$
 69 direction and an electric field in the $-z$ direction of
 70 appropriate magnitudes. The molecules therefore experience
 71 no torque and pass through a second horizontal molecular
 72 polarizer P2 and leave the field region. On the other hand, if
 73 we reverse the direction of \vec{v} and send molecules backward
 74 through P2, then there will be a torque on them (specifically
 75 in the $-x$ direction if \vec{p} is initially in the $+y$ direction). By
 76 suitable choice of the spacing between the two polarizers, we
 77 can arrange for the dipoles to be rotated by exactly 90° when
 78 they reach P1 and therefore be rejected by it.
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82

83 **FIGURE 1.** Sketch of a molecular isolator. Two horizontal
 84 molecular polarizers P1 and P2 sandwich a region of crossed
 85 magnetic and electric fields directed along the $+y$ and $-z$ axes,
 86 respectively. A long molecule with an electric dipole moment
 87 oriented along its axis is shown entering the device with a
 88 translational velocity in the $+x$ direction.
 89

90 Note that the resulting collision of the molecules with P1
 91 should be designed to be inelastic, with the rejected
 92 molecules falling vertically into some collection chamber

1 below the field region.¹ If the molecules instead reflected
2 elastically off P1, their rotational inertia would cause them to
3 continue to rotate as they traveled back toward P2. But since
4 they experience no torque on that return trip, they would
5 rotate by more than 90° and overshoot the acceptance slit in
6 P2 (by an angle of $180^\circ / \sqrt{2} - 90^\circ \approx 37^\circ$). If the molecules
7 reflected elastically off P2 also, they would then be returned
8 to P1 once again and this time be transmitted by it. In the
9 absence of losses, it is impossible to get a net flow of
10 molecules from the entrance to the exit side of the isolator.
11 Otherwise one would have a Maxwell's demon which, when
12 connected to two chambers of molecules, would maintain a
13 steady-state pressure imbalance between them. The optical
14 analog would be a nonabsorbing valve that permits the flow
15 of radiation in only one direction, creating a permanent
16 temperature imbalance between two connected chambers, in
17 violation of the second law of thermodynamics [11].
18

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23
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¹Other practical considerations are that the molecules need to be:
low density to avoid collisions [6], rotationally cooled to below 1 K
[7, 8] to minimize thermal reorientations, and traveling at high
speed (above 1 km/s [9]) to keep the magnetic field strength
reasonable (say 0.5 T [10]).