Standing Human - an Inverted Pendulum

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Abstract
A standing posture looks like an inverted pendulum, and its stabilization mechanism has been catching scientists' attention for ages. In this note, the posture stability is delineated at the lowest level-mechanical model. The foot sole is simplified as a twisted spring to stable the posture. The critical spring coefficient is derived.

Keywords: Inverted pendulum; Posture, Stability.

Resumen
Una postura de pie se ve como un péndulo invertido, y su mecanismo de estabilización ha llamado la atención de científicos por años. En esta nota, la estabilidad de la postura está determinada en el nivel más bajo de modelo mecánico. La planta del pie es simplificada como un resorte torcido para una postura estable. Derivamos el coeficiente crítico del resorte

Palabras clave: Péndulo invertido, Postura, Estabilidad.

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I. INTRODUCTION

A standing human looks like an inverted pendulum with the gravity center well above the ground [1]. The mechanism to keep the standing posture has intrigued scientists from several fields for a long time [2]. The standing posture stability has much significance in clinics. Stevens had reported that the direct medical cost incurred with falling of patients or aged people was up to 19 billion dollars in 2000 in the USA [3]. The following factors can influence the human standing posture directly or indirectly, Parkinson's Disease [4]、diabetes mellitus [5]、Ménière's disease [6]、stroke [7]、injured spinal cord [8], large dose of pyridoxine [9], etc. Therefore, research on the stance posture becomes an important branch of neuroscience, and its functioning units include muscles, skeletons, and peripheral sensors to the central neural system [10, 11, 12]. The state of art has entered into the gene level [13]. Its mechanism also has important value in selecting and training astronauts [14], sports athletes [15], and robotic design [16]. In addition, it aids to understand how the human eventually evolve to a stance posture [17].

In this note, we give a possible explanation at the simplest mechanical level using the inverted pendulum model, which provides an enticing dynamics example. The outline of this note is as follows, the physical model is shown in Sec II; the mathematical model is discussed in Sec III; and finally the conclusions are reported in Sec. IV.

II. PHYSICAL MODEL

At the mechanical level, a standing human can be simplified as a standing rigid body with a mass *m*, the mass center *C*, the mass center height above the ground l_c, and the rotational inertia J_C with respect to the mass center *C* (FIGURE 1).
The stabilization of an inverted pendulum can be achieved by exerting the pivot with high-frequency vertical oscillations [18-20]. Clearly, such mechanisms cannot be applied to a standing human.

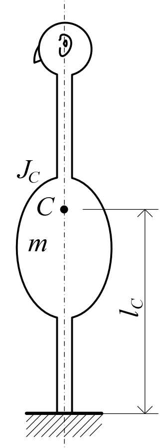

FIGURE 1. The model of a standing human.

The feet of a standing human have plate-like soles, rather than simple pivots. These plate-like soles are essential to stabilizing a standing posture. As an extreme counterexample (FIGURE 2) is the cruel three thousand years practice of foot wrapping of females in China until beginning of the 20th Century that made a woman totter while walking.

FIGURE 2. Wrapped feet with matched shoes.

At the mechanical level, a normal plate-like sole can be simplified as a coiled spring (coefficient *k*), as shown in FIGURE 3. The coiled spring plays a key role in maintaining stability, as does the plate-like sole when the human tilts.

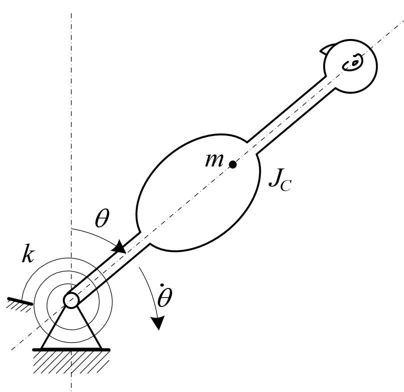

FIGURE 3. Inverted pendulum model with a coiled spring.

III. DERIVATION

We will investigate the model in FIGURE 3 by way of the energy method. Assume the coiled spring has an original length corresponding to an initial angle θ_0. The human when aligned vertically is given the status of zero potential energy. When the model tilts by the angle θ, the potential contains two terms: that of the spring resisting the movement (V_k) and that of gravity accelerating the rotation (V_h). They are

$$V_k = \frac{k}{2}[(\theta-\theta_0)^2-(0-\theta_0)^2],\ V_h = mgl_C(\cos\theta-1). \quad (1)$$

The total potential is

$$V = \frac{k}{2}[(\theta-\theta_0)^2-\theta_0^2]+mgl_c(\cos\theta-1). \quad (2)$$

The kinetic energy of a tilting human is

$$T = \frac{1}{2}(ml_C^2+J_C)\dot{\theta}^2. \quad (3)$$

In light of energy conservation, $\mathrm{d}(T+V)/\mathrm{d}t=0$, we have

$$\frac{\mathrm{d}t}{\mathrm{d}}\left\{\frac{k}{2}[(\theta-\theta_0)^2-\theta_0^2]+mgl_C(\cos\theta-1)+\frac{1}{2}(ml_C^2+J_C)\dot{\theta}^2\right\}=0. \quad (4)$$

It can derived that

$$k(\theta-\theta_0)\dot{\theta}-mgl_C\sin\theta\dot{\theta}+(ml_C^2+J_C)\ddot{\theta}\dot{\theta}=0. \quad (5)$$

Eliminating $\dot{\theta}$ leads to

$$(ml_C^2+J_C)\ddot{\theta}+F(\theta)=0, \quad (6)$$

where the force $F(\theta)$

$$F(\theta)=k(\theta-\theta_0)-mgl_C\sin\theta. \quad (7)$$

The angle θ_b corresponding to the balance center is as in

$$F(\theta_b)=0=k(\theta_b-\theta_0)-mgl_C\sin\theta_b. \quad (8)$$

That is

$$\theta_b-\frac{mgl_C}{k}\sin\theta_b-\theta_0=0\,. \quad (9)$$

Usually a small θ_b, corresponding to vertical alignment approximately, is concerned. In this case, $|\theta_b|\geq|\theta_0|$ can be derived from Eq. (9). This is because the human body weight pulls the initial balance angle θ_0 down to a new position θ_b.

We are most interested in the condition $\theta_b=0$, meaning that the balance status is the normal vertical alignment as a stance. Clearly, this requires $\theta_0=0$ according to Eq. (9). This indicates that the initial length of the coiled spring should be aligned vertically at first. However, this is not enough, because the balance center should be stable.

We introduce a new variable $\varphi=\theta-\theta_b$. Substituting $\theta=\varphi+\theta_b$ back to Eq. (7) leads to

$$(ml_C^2 + J_C)\ddot{\varphi} + F(\theta_b + \varphi) = 0\,. \tag{10}$$

For stability analysis, φ is assumed to be small, thus $F(\varphi+\theta_b)$ is expanded as the Taylor series around φ=0. That is

$$F(\theta_b + \varphi) = F(\theta_b) + (k - mgl_C \cos\theta_b)\varphi + \frac{F''(\theta_b)}{2}\varphi^2 + \frac{F''(\theta_b)}{6}\varphi^3 + ... \tag{11}$$

On the right-hand side, the first term is zero because of Eq. (8). The term higher with orders than φ^2 can be ignored since φ is small. Accordingly,

$$(ml_C^2 + J_C)\ddot{\varphi} + (k - mgl_C \cos\theta_b)\,\varphi = 0\,. \tag{12}$$

Hence, a stable θ_b requires that

$$k > mgl_C \cos\theta_b\,. \tag{13}$$

Specifically, for the most concerned case θ_b=0 (vertical alignment), we have

$$k > mgl_C\,. \tag{14}$$

Eq. (14) indicates that in order to have a stable stance, the torque generated by the twist spring must be greater than that produced by the gravity.

θ_b=180 means the foot was hung over, the equivalent stiffness in Eq. (12) is always positive. That is to say, this is always a stable posture.

Of course, a significant higher k makes the standing posture stable, but it also may cost more biological energy, in addition to losing movement flexibility. If the twist spring has the critical value k=mgl_c, the potential in Eq. (2) reduces

$$V = mgl_c(\cos\theta - 1 + \theta^2/2) > 0 \text{ for a small } \theta\,. \tag{15}$$

Consequently θ_b=0 is still a stable balance center. In this case, it is the nonlinear term of the gravity potential that stabilizes the stance.

IV. CONCLUSIONS

The standing posture was discussed based on the simplest mechanical level using the model of an inverted pendulum with coiled spring, and the critical spring coefficient to keep a stable posture was derived. It was shown that the coiled spring (equivalent to the sole's function) must be adapted (might be driven by the evolutional force) to be aligned vertically at first to keep a vertical posture efficiently. It was also shown that for critical spring coefficient, it is the nonlinear term of the gravity potential that stabilizes the stance.

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