Photosynthetic Engine over the Globe

EDVCATIO PHYSICORVM
SIC ITUR AD ASTRA
QVO NON ASCENDAM?

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Abstract

According to M King Hubbert, a pioneer on fuel reserves, on an annual basis about 1.26x10^{21}J of solar energy in the photosynthetically active region goes for photosynthesis over the globe whereas the harvested value as reported by Food & Agriculture Organization is more than two times of this estimate. This paper derives the value of the upper limit which can be harvested under ideal conditions and it is shown that at present the world consumes only 0.1% of biomass that can be so produced. This gives hope that the growing huge population of world will not starve.

Keywords: Solar energy, photosynthetic engine, harvested value, upper limit.

Resumen

De acuerdo con M. King Hubbert, un pionero en las reservas de combustible, sobre una base anual sobre 1.26x1021J de la energía solar en la región fotosintéticamente activa se va para la fotosíntesis en el mundo entero mientras que el valor recolectado según lo informado por la Organización de Alimentación y Agricultura es más de dos veces de esta estimación. Este artículo deriva el valor del límite superior que pueden ser cosechados en condiciones ideales y se demuestra que en la actualidad el mundo consume sólo el 0,1% de la biomasa que pueden ser producida de ese modo. Esto da esperanza de que la enorme población creciente del mundo no se morirá de hambre.

Palabras claves: La energía solar, el motor de fotosíntesis, el valor de la cosecha, el límite superior.

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I. INTRODUCCTION

Studying, measuring, and estimating the solar energy which can be harvested by photosynthesis is of paramount importance for the purpose of feeding the huge growing population on the earth. Its value was quoted by M. King Hubbert [1], a pioneer on fuel reserves, as $P_P = 40x10^{12}$ J s^{-1} out of the total solar radiation $Q = 174000x10^{12}$ J s^{-1} being intercepted on the average by half of the globe. On the annual basis this corresponds to the harvested solar energy by photosynthesis as $P_P^A = 3.15x10^7 P_P = 1.26x10^{21}$ J y^{-1} producing a biomass equivalent to $6.45x10^{-8} P_P^A = 8.13x10^{13}$ Kg y^{-1}. The reported harvested value of solar energy by plants and photosynthetic organism is $3x10^{21}$ J y^{-1} equivalent to a biomass of $19.4x10^{13}$ Kg y^{-1}. Thus the achieved harvested value of solar energy is more than two times that projected by King. This clearly shows that the upper limit of solar energy which may be utilized by photosynthesis under ideal conditions should be higher by couple of order of magnitude. This also gives a scope to enhance the production of crop to meet the requirement [2] of the increasing population of the globe. The aim of the present comment is to address to this question for the benefit of students of physics.

II. FORMULATION

The solar energy is electromagnetic in nature which is characterized by wavelength λ, frequency v and velocity c satisfying the relation

$$c = \lambda \nu, 0 \le \lambda \le \infty, \infty \le \nu \le 0 \qquad (1)$$

The electromagnetic spectrum [3, 4] extends from below the radio frequencies at the long-wavelength end through gamma radiation at the short-wavelength end covering wavelengths from thousands of kilometers down to a fraction of the size of an atom. Assuming that the Sun has as a uniform temperature T over its surface the Planck's radiation law [4, 5] says that

$$I(\lambda,T)d\lambda = \frac{\varepsilon(\lambda,T)A(2\pi hc^2)d\lambda}{\lambda^5\{\exp(hc/\lambda kT)-1\}} \text{ J s}^{-1}. \qquad (2)$$

$I(\lambda,T)d\lambda$ is the power radiated between the wavelengths λ and $\lambda + d\lambda$, A is the surface area, ε is the emissivity and the constants h and k, respectively are Planck's constant and Boltzmann's constant. For simplicity, considering the Sun to be an ideal blackbody ($\varepsilon = 1$) the solar flux Q emitted over all the wavelengths from the unit area ($A = 1$ m^2) of the Sun is

$$Q = \int_0^\infty I(\lambda, T) d\lambda = \sigma T^4 \text{ Jm}^{-2}\text{s}^{-1}, \tag{3}$$

where σ is the Stefan-Boltzmann constant. When this flux reaches the earth this is diluted by a factor [6]

$$f = \frac{R_S^2}{d^2} \cdot \tag{4}$$

Here R_S is the radius of the Sun and d is the yearly mean distance between the earth and the Sun. Now the calculation of the value of solar flux in between wavelengths λ_i and λ_f present in the solar radiation will be taken up.

A. Expression of solar flux in the region λ_i and λ_f

The photosynthetically active region (PAR) $\lambda_i = 400$ nm to $\lambda_f = 750$ nm is responsible for the process of photosynthesis [7] in plants and algae as shown in figure 1 where the light absorption (in percent) for the pigments chlorophyll, carotenoids and phycocyanin are depicted.

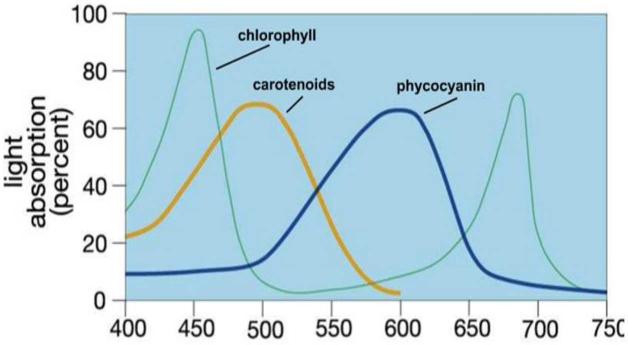

FIGURE 1. Plot of light absorption in percentage for chlorophyll, carotenoids and phycocynanin against the wavelength of solar radiation.

The collective curve corresponding to these three pigments over the said wavelengths region can be displayed as shown [7] in figure 2 enclosing the ABCDE shaded area in green. It is clear that the light absorption is not uniform over the region and this absorption factor will be denoted by $\alpha(\lambda)$ % in the sequel. The expression for the solar flux $Q(\lambda_i \to \lambda_f)$ emitted from an unit area of the Sun in between wavelengths λ_i and λ_f according to (3) would be

$$Q(\lambda_i \to \lambda_f) = \int_{\lambda_i}^{\lambda_f} I(\lambda, T).d\lambda \text{ Jm}^{-2}\text{s}^{-1}. \tag{5}$$

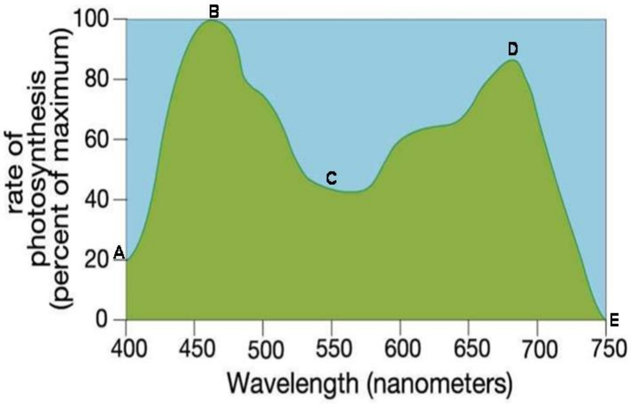

FIGURE 2. The collective rate of photosynthesis curve for the three pigments against the wavelength shown in figure 1.

B. Solar Flux into Photosynthetic channel

This flux will be diluted by the factor f [*cf*. Eq. (4)] when it reaches the earth's atmosphere. Also, multiplying this by the absorption factor $\alpha(\lambda)$ the amount of solar energy going into the channel of photosynthesis on half of the earth will be

$$Q_P = \int_{\lambda_i}^{\lambda_f} \frac{2\pi hc^2 d\lambda}{\lambda^5\{\exp(hc/\lambda kT) - 1\}} \alpha(\lambda).f.\pi R_E^2 \text{ J.s}^{-1}. \tag{6}$$

Here R_E is the radius of the earth. On the annual basis the solar energy going into the photosynthetic channel will be

$$Q_P^A = 3.15x10^7 Q_P \text{ J y}^{-1}; \text{ 1 y} = 3.15\text{x}10^7 \text{ s} . \tag{7}$$

C. Efficiency and Biomass Production

The photosynthetic engine also works like any other engine in accordance with the relation

$$OutputPower(P_P^A) = Efficiency(\eta) x InputPower(Q_P^A) . \tag{8}$$

In the present case the absorbed solar energy in the photosynthetically active region corresponds to

$$InputPower = Q_P^A \text{ J y}^{-1}. \tag{9}$$

If the efficiency of the photosynthetic engine is η then the actual energy being utilized in the production of biomass and the value of biomass so produced would be

$$OutputPower = P_P^A = \eta Q_P^A \text{ J y}^{-1}, \tag{10}$$

$$M_P^A = 6.45x10^{-8} P_P^A \text{ Kg y}^{-1} \qquad (11)$$

The factor [6] 6.45x10^{-8} Kg J^{-1} corresponds to production of biomass 6.45x10^{-8} Kg for every Joule of photosynthetically active radiation being utilized.

D. Parameterization of absorption factor $\alpha(\lambda')$

The parameterization of the absorption factor $\alpha(\lambda')$ present in the integrand of Eq.(6) is approximated by dividing the rate of photosynthesis curve of figure 2 in four blocks with the straight lines AB, BC, CD, and DE [*cf.* Fig. 3] whose equations are, respectively

1. Line AB: $\alpha(\lambda') = 1.2698\lambda' - 487.9\,\%$; $400 \le \lambda' < 463$
2. Line BC: $\alpha(\lambda') = -0.6437\lambda' + 398.0\,\%$; $463 \le \lambda' < 550$
3. Line CD: $\alpha(\lambda') = 0.3231\lambda' - 133.7\,\%$; $550 \le \lambda' < 680$
4. Line DE : $\alpha(\lambda') = -1.2285\lambda' + 921.4\,\%$; $680 \le \lambda' < 750$ (12)

Here λ' satisfies the relation $\lambda = 10^{-9}\lambda'$m and the coordinates of the points A, B, C, D, and E, respectively being (400, 20), (463,100), (550, 44), (680, 86), and (750, 0). Now we turn to numerical work.

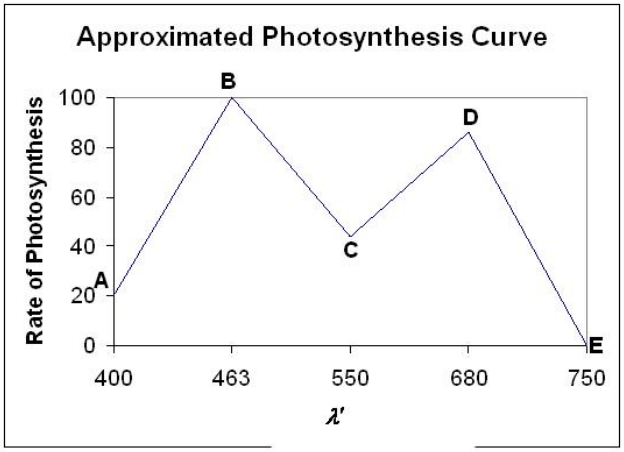

FIGURE 3. The straight line approximation of rate of photosynthesis curve shown in figure 2.

III. NUMERICAL WORK

The analytical solution of the integral in Eq. (6) is not possible and therefore its value was obtained using Simpson's quadrature and employing [4, 8]

$$h = 6.63x10^{-34}\,Js : c = 3.0x10^{8}\,m/s; k = 1.38x10^{-23}\,J/K$$
$$R_S = 6.96\,x10^{8}\,m; R_E = 6.37\,x10^{6}\,m; d = 1.50\,x10^{11}\,m : T = 5776\,K \qquad (13)$$
$$\lambda_i = 400\,x10^{-9}\,m\,,\ \lambda_f = 750\,x10^{-9}\,m$$

as well as relations (4), (7) and (12). The calculated value of solar energy going into the photosynthetic channel per annum Q_P^A [*cf.* Eq. (7, 9)] is listed (row 2, column 3) in Table I.

TABLE I. Reported and estimated values of solar energy which can be harvested by photosynthesis.

Particular	Reported value	Estimated optimum value (present work)
Input solar energy into photosynthetic channel Q_P^A	-	1.4x10^{24} J/y
Efficiency η	Typical Plants [7] ~0.1% Typical crop plants [9] ~0.2-2% Sea plants algae [11] ~ 18%	20% [9]
Output /harvested solar energy $P_P^A = \eta Q_P^A$	1. Solar Energy [1] ~1.26x10^{21}J/y; Equivalent Biomass ~8.13x10^{13}Kg/y 2. Solar Energy [10,12] ~3x10^{21} J/y, Equivalent Biomass [10,12]~ 1.94x10^{14} Kg/y	Solar energy ~ 2.8x10^{23} J/y Equivalent Biomass ~ 1.8x10^{16} Kg/y

The upper theoretical limit of the efficiency of the photosynthetic channel reported in the literature [9] is

$$\eta = 20\%\,. \qquad (14)$$

The above efficiency provides an estimate for maximum possible utilization of solar energy under ideal conditions through photosynthesis [*cf.* Eq. (10)] and the corresponding equivalent biomass [*cf.* Eq. (11)] (see row 4, column 3 of Table I). The corresponding numbers reported by King and the reported biomass harvested by the mankind are also listed in Table I (row 4, column 2).

IV. CONCLUSIONS & DISCUSSION

The salient conclusions of the present work are discussed below.

- If the process of photosynthesis works as an engine then the input power in the form of solar energy in the photosynthetically active region 400 nm to 750 nm on annual basis to this engine is 1.4x10^{24} J y^{-1}.
- The theoretical upper limit [9] on the efficiency of this engine being 20% the output power that can be achieved under ideal conditions would be 2.8x10^{23} J y^{-1}. This is around 200 times more than

the value projected by M. King Hubbert [1] (1.26×10^{21}.J y^{-1}).

- The theoretical upper limit on the equivalent biomass harvested would be 1.8×10^{16} Kg y^{-1} against the achieved biomass production [10] so far being 2×10^{14} Kg y^{-1}.
- The amounts consumed [10] by human beings are quite small (3×10^{20}J y^{-1} or 2×10^{13}Kg y^{-1}) representing only 10% of the achieved production through photosynthesis.
- The consumed part is marginal (~0.1%) in comparison to the theoretical upper limit that could be harvested. This observation gives hope that the growing huge population of world will not starve.

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